Solution to HW #2

Please be advised that for problem 18-2, the solution used machined surface factor instead of the HR surface factor. Theoretically most of the shafts could be assumed machined since we generally turn the shoulders and other mounting grooves on the stock material which could be CD or HR.

18-2. This will be solved using a deterministic approach with \( n_d = 2 \). However, the reader may wish to explore the stochastic approach given in Sec. 7-17.

Table A-20: \( S_{ut} = 68 \text{ kpsi} \) and \( S_y = 37.5 \text{ kpsi} \)

Eq. (7-8): \( S'_c = 0.504(68) = 34.27 \text{ kpsi} \)

Eq. (7-18): \( k_a = 2.70(68)^{-0.265} = 0.883 \)

Assume a shaft diameter of 1.8 in.

Eq. (7-19):
\[
k_b = \left( \frac{1.8}{0.30} \right)^{0.107} = 0.826
\]

\( k_c = k_d = k_f = 1 \)

From Table 7-7 for \( R = 0.999, k_e = 0.753 \).

Eq. (7-17): \( S_c = 0.883(0.826)(1)(1)(1)(0.753)(34.27) = 18.8 \text{ kpsi} \)

From p. 444, \( K_t = 2.14, \ K_{ts} = 2.62 \)

With \( r = 0.02 \text{ in} \) Figs. 7-20 and 7-21 give \( q = 0.60 \) and \( q_s = 0.77 \), respectively.

Eq. (7-31):
\[
K_f = 1 + 0.60(2.14 - 1) = 1.68
\]
\[
K_{fs} = 1 + 0.77(2.62 - 1) = 2.25
\]

(a) DE-elliptic from Eq. (18-21),

\[
d = \left\{ \frac{16(2)}{\pi} \left[ 4 \left( \frac{1.68(2371)}{18 800} \right)^2 + 3 \left( \frac{2.25(2626)}{37 500} \right)^2 \right]^{1/2} \right\}^{1/3} \approx 1.725 \text{ in } \text{Ans.}
\]

(b) DE-Gerber from Eq. (18-16),

\[
d = \left[ \frac{16(2)(1.68)(2371)}{\pi(18 800)} \left\{ 1 + 3 \left( \frac{2.25(2626)(18 800)}{1.68(2371)(68 000)} \right)^2 \right\}^{1/2} \right]^{1/3}
\]

\( = 1.687 \text{ in } \text{Ans.} \)

From Prob. 18-1, deflection controls \( d = 1.81 \text{ in} \)
18-8 From the shaft forces in Probs. 18-1 and 18-7 solutions, we can construct the force portions of the free-body diagram for shaft $b$ of Fig. P18-1.

\[ T = 63.025 \left( \frac{50}{240} \right) = 13.130 \text{ lbf \cdot in} \]
The resulting forces at each location are not in the same plane; therefore, we must work in terms of components.

\[
d = \left| \frac{32(2)}{3\pi(30)(10^6)(11)(0.001)} \left\{ \left[ 1531(8)(2^2 - 11^2) - 1313(2)(2^2 - 11^2) \right]^2 \\
+ \left[ 3283(8)(2^2 - 11^2) - 612(2)(2^2 - 11^2) \right]^2 \right\}^{1/2} \right|
\]

\[= 2.320 \text{ in}\]

with \(d = 2.320\), determine the slopes at \(C, G, H,\) and \(D\).

\[
\theta_C = \frac{1}{6(30)(10^6)(\pi/64)(2.320^4)(11)} \left\{ \left[ 1531(8)(2^2 - 11^2) - 1313(2)(2^2 - 11^2) \right]^2 \\
+ \left[ 3283(8)(2^2 - 11^2) - 612(2)(2^2 - 11^2) \right]^2 \right\}^{1/2}
\]

\[= 0.000500 \text{ rad} \quad \text{(checks)}\]

\[
\theta_G = \frac{1}{6(30)(10^6)(\pi/64)(2.320^4)(11)} \left\{ \left[ 1531(8)(3(3^2) + 8^2 - 11^2) \\
- 1313(2)(3(3^2) + 2^2 - 11^2) \right]^2 \\
+ \left[ 3283(8)(3(3^2) + 8^2 - 11^2) \\
- 612(2)(3(3^2) + 2^2 - 11^2) \right]^2 \right\}^{1/2}
\]

\[= 0.000245 \text{ rad}\]

\[
\theta_H = \frac{1}{6(30)(10^6)(\pi/64)(2.320^4)(11)} \left\{ \left[ 1531(3)(11^2 - 3(2^2) - 3^2) \\
- 1313(9)(11^2 - 3(2^2) - 9^2) \right]^2 \\
+ \left[ 3283(3)(11^2 - 3(2^2) - 3^2) \\
- 612(9)(11^2 - 3(2^2) - 9^2) \right]^2 \right\}^{1/2}
\]

\[= 0.000298 \text{ rad}\]

\[
\theta_D = \frac{1}{6(30)(10^6)(\pi/64)(2.320^4)(11)} \left\{ \left[ 1531(3)(11^2 - 3^2) \\
- 1313(9)(11^2 - 9^2) \right]^2 \\
+ \left[ 3283(3)(11^2 - 3^2) \\
- 612(9)(11^2 - 9^2) \right]^2 \right\}^{1/2}
\]

\[= 0.000314 \text{ rad}\]

The shaft diameter should be increased for the same reason given in Problem 18-4 (gear mesh slope).

\[
d_{new} = 2.32 \left| \frac{20(0.000298)}{0.00025} \right|^{1/4} = 2.883 \text{ in}
\]
Strength constraints

\[ M_G = 3\sqrt{875^2 + 2276^2} = 7315 \text{ lbf \cdot in} \]
\[ M_H = 2\sqrt{657^2 + 395^2} = 1533 \text{ lbf \cdot in} \]

Point G is more critical. Assume \( d = 2.88 \text{ in} \).

Similar to Prob. 8-2, \( S_{ul} = 68 \text{ kpsi}, \quad S_y = 37.5 \text{ kpsi} \quad \text{and} \quad k_d = 0.883 \)

Eq. (7-19):
\[ k_b = 0.91(2.88)^{-0.157} = 0.771 \]
\[ k_c = k_d = k_f = 1 \]

For \( R = 0.995 \), Table A-10 provides \( z = 2.576 \).

Eq. (7-28):
\[ k_e = 1 - 0.08(2.576) = 0.794 \]

Eq. (7-17):
\[ S_e = 0.883(0.771)(1)(1)(0.794)(1)(0.504)(68) = 18.5 \text{ kpsi} \]

Eq. (7-31):
\[ K_f = 1.68, \quad K_{fs} = 2.25 \]

Using DE-elliptic theory, Eq. (18-21)

\[ d = \left\{ \frac{16(2)}{\pi} \left[ 4 \left( \frac{1.68(7315)}{18 \quad 500} \right)^2 + 3 \left( \frac{2.25(13 \quad 130)}{37 \quad 500} \right)^2 \right]^{1/2} \right\}^{1/3} \]
\[ = 2.687 \text{ in} \quad O.K. \]

Students will approach the design differently from this point on.