Heat is transferred to a piston-cylinder device with a set of stops. The work done, the heat transfer, the exergy destroyed, and the second-law efficiency are to be determined.

**Assumptions**
1. The device is stationary and kinetic and potential energy changes are zero.
2. There is no friction between the piston and the cylinder.

**Analysis**

(a) The properties of the refrigerant at the initial and final states are (Tables A-11 through A-13)

\[
\begin{align*}
P_1 &= 120 \text{ kPa} & \nu_1 &= 0.16544 \text{ m}^3/\text{kg} \\
T_1 &= 20^\circ \text{C} & u_1 &= 248.22 \text{ kJ/kg} \\
& & s_1 &= 1.0624 \text{ kJ/kg.K} \\
P_2 &= 180 \text{ kPa} & \nu_2 &= 0.17563 \text{ m}^3/\text{kg} \\
T_2 &= 120^\circ \text{C} & u_2 &= 331.96 \text{ kJ/kg} \\
& & s_2 &= 1.3118 \text{ kJ/kg.K}
\end{align*}
\]

The boundary work is determined to be

\[
W_{b,\text{out}} = mP_2(\nu_2 - \nu_1) = (1.4 \text{ kg})(180 \text{ kPa})(0.17563 - 0.16544)\text{m}^3/\text{kg} = 2.57 \text{ kJ}
\]

(b) The heat transfer can be determined from an energy balance on the system

\[
Q_{\text{in}} = m(u_2 - u_1) + W_{b,\text{out}} = (1.4 \text{ kg})(331.96 - 248.22)\text{kJ/kg} + 2.57 \text{ kJ} = 119.8 \text{ kJ}
\]

(c) The exergy difference between the inlet and exit states is

\[
\Delta X = m[u_2 - u_1 - T_0(s_2 - s_1) + P_0(\nu_2 - \nu_1)]
\]

\[
= (1.4 \text{ kg})[(331.96 - 248.22)\text{kJ/kg} - (298 \text{ K})(1.3118 - 1.0624)\text{kg.K} + (100 \text{ kPa})(0.17563 - 0.16544)\text{m}^3/\text{kg}]
\]

\[
= 14.61 \text{ kJ}
\]

The useful work output for the process is

\[
W_{u,\text{out}} = W_{b,\text{out}} - mP_0(\nu_2 - \nu_1) = 2.57 \text{ kJ} - (1.4 \text{ kg})(100 \text{ kPa})(0.17563 - 0.16544)\text{m}^3/\text{kg} = 1.14 \text{ kJ}
\]

The exergy destroyed is the difference between the exergy difference and the useful work output

\[
X_{\text{dest}} = \Delta X - W_{u,\text{out}} = 14.61 - 1.14 = 13.47 \text{ kJ}
\]

(d) The second-law efficiency for this process is

\[
\eta_\Pi = \frac{W_{u,\text{out}}}{\Delta X} = \frac{1.14 \text{ kJ}}{14.61 \text{ kJ}} = 0.078
\]
Steam expands in a turbine, which is not insulated. The reversible power, the exergy destroyed, the second-law efficiency, and the possible increase in the turbine power if the turbine is well insulated are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Potential energy change is negligible.

**Analysis**

(a) The properties of the steam at the inlet and exit of the turbine are (Tables A-4 through A-6)

\[
\begin{align*}
P_1 &= 12 \text{ MPa} \quad h_1 = 3481.7 \text{ kJ/kg} \\
T_1 &= 550^\circ\text{C} \quad s_1 = 6.6554 \text{ kJ/kg.K} \\
P_2 &= 20 \text{ kPa} \quad h_2 = 2491.1 \text{ kJ/kg} \\
x_2 &= 0.95 \quad s_2 = 7.5535 \text{ kJ/kg.K}
\end{align*}
\]

The enthalpy at the dead state is

\[
h_0 = 104.83 \text{ kJ/kg}
\]

The mass flow rate of steam may be determined from an energy balance on the turbine

\[
m\left( h_1 + \frac{V_1^2}{2} \right) = m\left( h_2 + \frac{V_2^2}{2} \right) + \dot{Q}_{\text{out}} + \dot{W}_a
\]

\[
m\left[ 3481.7 \text{ kJ/kg} + \frac{(60 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \right] = m\left[ 2491.1 \text{ kJ/kg} + \frac{(130 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \right] + 150 \text{ kW} + 2500 \text{ kW}
\]

\[
\dot{m} = 2.693 \text{ kg/s}
\]

The reversible power may be determined from

\[
\dot{W}_{\text{rev}} = m\left( h_1 - h_2 - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} \right)
\]

\[
= (2.693) \left[ (3481.7 - 2491.1) - (298)(6.6554 - 7.5535) + \frac{(60 \text{ m/s})^2 - (130 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \right] = 3371 \text{ kW}
\]

(b) The exergy destroyed in the turbine is

\[
\dot{X}_{\text{dest}} = \dot{W}_{\text{rev}} - \dot{W}_a = 3371 - 2500 = 871 \text{ kW}
\]

(c) The second-law efficiency is

\[
\eta_{II} = \frac{\dot{W}_a}{\dot{W}_{\text{rev}}} = \frac{2500 \text{ kW}}{3371 \text{ kW}} = 0.742
\]

(d) The energy of the steam at the turbine inlet in the given dead state is

\[
\dot{Q} = \dot{m}(h_1 - h_0) = (2.693 \text{ kg/s})(3481.7 - 104.83) \text{ kJ/kg} = 9095 \text{ kW}
\]

The fraction of energy at the turbine inlet that is converted to power is

\[
f = \frac{\dot{W}_a}{\dot{Q}} = \frac{2500 \text{ kW}}{9095 \text{ kW}} = 0.2749
\]

Assuming that the same fraction of heat loss from the turbine could have been converted to work, the possible increase in the power if the turbine is to be well-insulated becomes

\[
\dot{W}_{\text{increase}} = f\dot{Q}_{\text{out}} = (0.2749)(150 \text{ kW}) = 41.2 \text{ kW}
\]
Argon gas is expanded adiabatically in an expansion valve. The exergy of argon at the inlet, the exergy destruction, and the second-law efficiency are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are zero. 3 Argon is an ideal gas with constant specific heats.

**Properties** The properties of argon gas are $R = 0.2081 \text{ kJ/kg.K}$, $c_p = 0.5203 \text{ kJ/kg.ºC}$ (Table A-2).

**Analysis**

(a) The exergy of the argon at the inlet is

$$x_1 = h_1 - h_0 - T_0 (s_1 - s_0)$$

$$= c_p(T_1 - T_0) - T_0 \left[ c_p \ln \frac{T_1}{T_0} - R \ln \frac{P_1}{P_0} \right]$$

$$= (0.5203 \text{ kJ/kg.K})(100 - 25)\ºC - (298 \text{ K}) \left[ (0.5203 \text{ kJ/kg.K})\ln \frac{373 \text{ K}}{298 \text{ K}} - (0.2081 \text{ kJ/kg.K})\ln \frac{3500 \text{ kPa}}{100 \text{ kPa}} \right]$$

$$= 224.7 \text{ kJ/kg}$$

(b) Noting that the temperature remains constant in a throttling process of an ideal gas, the exergy destruction is determined from

$$x_{\text{dest}} = T_0 s_{\text{gen}}$$

$$= T_0 (s_2 - s_1)$$

$$= T_0 \left[ -R \ln \frac{P_1}{P_0} \right] = (298 \text{ K}) \left[ - (0.2081 \text{ kJ/kg.K})\ln \frac{500 \text{ kPa}}{3500 \text{ kPa}} \right]$$

$$= 120.7 \text{ kJ/kg}$$

(c) The second-law efficiency is

$$\eta_{II} = \frac{x_1 - x_{\text{dest}}}{x_1} = \frac{(224.7 - 120.7) \text{ kJ/kg}}{224.7 \text{ kJ/kg}} = 0.463$$
A geothermal heat pump is considered. The degrees of subcooling done on the refrigerant in the condenser, the mass flow rate of the refrigerant, the heating load, the COP of the heat pump, the minimum power input are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) From the refrigerant-134a tables (Tables A-11 through A-13)

\[
\begin{align*}
T_4 &= 20^\circ C \\ P_4 &= 572.1 \text{kPa} \\
x_4 &= 0.23 \\ h_4 &= 121.24 \text{kJ/kg} \\
h_3 &= h_4 \\
P_1 &= 572.1 \text{kPa} \\ h_1 &= 261.59 \text{kJ/kg} \\
x_1 &= 1 \text{(sat. vap.)} \\ s_1 &= 0.9223 \text{kJ/kg} \\
P_2 &= 1400 \text{kPa} \\ h_2 &= 280.00 \text{kJ/kg} \\
s_2 &= s_1
\end{align*}
\]

From the steam tables (Table A-4)

\[
\begin{align*}
h_{w1} &= h_f @ 50^\circ C = 209.34 \text{kJ/kg} \\
h_{w2} &= h_f @ 40^\circ C = 167.53 \text{kJ/kg}
\end{align*}
\]

The saturation temperature at the condenser pressure of 1400 kPa and the actual temperature at the condenser outlet are

\[
T_{\text{sat} @ 1400 \text{kPa}} = 52.40^\circ C
\]

\[
P_3 = 1400 \text{ kPa} \\
h_3 = 121.24 \text{kJ/kg}
\]

Then, the degrees of subcooling is

\[
\Delta T_{\text{subcool}} = T_{\text{sat} @ 1400 \text{kPa}} - T_3 = 52.40 - 48.59 = 3.81^\circ C
\]

(b) The rate of heat absorbed from the geothermal water in the evaporator is

\[
\dot{Q}_L = \dot{m}_w(h_{w1} - h_{w2}) = (0.065 \text{ kg/s})(209.34 - 167.53)\text{kJ/kg} = 2.718 \text{ kW}
\]

This heat is absorbed by the refrigerant in the evaporator

\[
\dot{m}_R = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{2.718 \text{ kW}}{(261.59 - 121.24)\text{kJ/kg}} = 0.01936 \text{ kg/s}
\]

(c) The power input to the compressor, the heating load and the COP are

\[
\dot{W}_\text{in} = \dot{m}_R(h_2 - h_1) + \dot{Q}_\text{out} = (0.01936 \text{ kg/s})(280.00 - 261.59)\text{kJ/kg} = 0.6564 \text{ kW}
\]

\[
\dot{Q}_H = \dot{m}_R(h_2 - h_3) = (0.01936 \text{ kg/s})(280.00 - 121.24)\text{kJ/kg} = 3.074 \text{ kW}
\]

\[
\text{COP} = \frac{\dot{Q}_H}{\dot{W}_\text{in}} = \frac{3.074 \text{ kW}}{0.6564 \text{ kW}} = 4.68
\]

(d) The reversible COP of the cycle is

\[
\text{COP}_{\text{rev}} = \frac{1}{1 - T_L / T_H} = \frac{1}{1 - (25 + 273)/(50 + 273)} = 12.92
\]

The corresponding minimum power input is

\[
\dot{W}_{\text{in, min}} = \frac{\dot{Q}_H}{\text{COP}_{\text{rev}}} = \frac{3.074 \text{ kW}}{12.92} = 0.238 \text{ kW}
\]
A gas refrigeration cycle with helium as the working fluid is considered. The minimum temperature in the cycle, the COP, and the mass flow rate of the helium are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Helium is an ideal gas with constant specific heats.
3. Kinetic and potential energy changes are negligible.

**Properties**
The properties of helium are $c_p = 5.1926 \text{kJ/kg} \cdot \text{K}$ and $k = 1.667$ (Table A-2).

**Analysis (a)** From the isentropic relations,

$$T_{2s} = T_1 \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = (263 \text{K})^{(3/1.667)} = 408.2 \text{K}$$

$$T_{4s} = T_3 \left( \frac{P_4}{P_3} \right)^{(k-1)/k} = (323 \text{K})^{(1/1.667)} = 208.1 \text{K}$$

and

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}} \rightarrow T_4 = T_3 \left( 1 - \eta_T \right) = 323 \left( 1 - 0.80 \right) = 231.1 \text{ K} = T_{\text{min}}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} \rightarrow T_2 = T_1 + \left( T_{2s} - T_1 \right) / \eta_C = 263 + \left( 408.2 - 263 \right) / 0.80 = 444.5 \text{ K}$$

**Analysis (b)** The COP of this gas refrigeration cycle is determined from

$$\text{COP}_R = \frac{q_L}{w_{\text{net,in}}} = \frac{q_L}{w_{\text{comp,in}} - w_{\text{turb,out}}} = \frac{h_3 - h_4}{(h_2 - h_1) - (h_3 - h_4)} = \frac{T_1 - T_4}{(T_2 - T_1) - (T_3 - T_4)} = \frac{263 - 231.1}{(444.5 - 263) - (323 - 231.1)} = 0.356$$

**Analysis (c)** The mass flow rate of helium is determined from

$$\dot{m} = \frac{\dot{Q}_{\text{refrig}}}{q_L} = \frac{\dot{Q}_{\text{refrig}}}{h_1 - h_4} = \frac{\dot{Q}_{\text{refrig}}}{c_p(T_1 - T_4)} = \frac{18 \text{ kJ/s}}{(5.1926 \text{ kJ/kg} \cdot \text{K})(263 - 231.1) \text{K}} = 0.109 \text{ kg/s}$$
A two-evaporator compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The cooling load of both evaporators per unit of flow through the compressor and the COP of the system are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

From the refrigerant tables (Tables A-11E, A-12E, and A-13E),

\[
\begin{align*}
\text{P}_3 &= 160 \text{ psia} \\
\text{sat. liquid} & \\
\text{h}_3 &= h_f \text{ @ 160 psia} = 48.519 \text{ Btu/lbm} \\
\text{h}_4 &= h_f \geq h_3 = 48.519 \text{ Btu/lbm} \quad \text{(throttling)} \\
T_5 &= 30^\circ \text{F} \\
\text{sat. vapor} & \\
\text{h}_5 &= h_g \text{ @ 30}^\circ \text{F} = 107.40 \text{ Btu/lbm} \\
T_7 &= -29.5^\circ \text{F} \\
\text{sat. vapor} & \\
\text{h}_7 &= h_g \text{ @ -29.5}^\circ \text{F} = 98.68 \text{ Btu/lbm} 
\end{align*}
\]

For a unit mass flowing through the compressor, the fraction of mass flowing through Evaporator II is denoted by \( x \) and that through Evaporator I is \( y \) \( (y = 1-x) \). From the cooling loads specification,

\[
\begin{align*}
\dot{Q}_{L,vap1} &= 2\dot{Q}_{L,vap2} \\
x(h_5 - h_4) &= 2y(h_7 - h_6)
\end{align*}
\]

where

\[
x = 1 - y
\]

Combining these results and solving for \( y \) gives

\[
y = \frac{h_5 - h_4}{2(h_7 - h_6) + (h_5 - h_4)} = \frac{107.40 - 48.519}{2(98.68 - 48.519) + (107.40 - 48.519)} = 0.3698
\]

Then,

\[
x = 1 - y = 1 - 0.3698 = 0.6302
\]

Applying an energy balance to the point in the system where the two evaporator streams are recombined gives

\[
xh_5 + yh_7 = h_1 \quad \rightarrow \quad h_1 = \frac{xh_5 + yh_7}{1} = \frac{(0.6302)(107.40) + (0.3698)(98.68)}{1} = 104.18 \text{ Btu/lbm}
\]

Then,
\[
P_1 = P_{\text{sat} (-29.5^\circ\text{F})} \approx 10 \text{ psia}
\]
\[
h_1 = 104.18 \text{ Btu/lbm}
\]
\[
P_2 = 160 \text{ psia}
\]
\[
h_2 = 131.14 \text{ Btu/lbm}
\]
The cooling load of both evaporators per unit mass through the compressor is
\[
q_L = x(h_5 - h_4) + y(h_7 - h_6)
\]
\[
= (0.6302)(107.40 - 48.519) \text{ Btu/lbm} + (0.3698)(98.68 - 48.519) \text{ Btu/lbm}
\]
\[
= 55.66 \text{ Btu/lbm}
\]
The work input to the compressor is
\[
w_{\text{in}} = h_2 - h_1 = (131.14 - 104.18) \text{ Btu/lbm} = 26.96 \text{ Btu/lbm}
\]
The COP of this refrigeration system is determined from its definition,
\[
\text{COP}_{R} = \frac{q_L}{w_{\text{in}}} = \frac{55.66 \text{ Btu/lbm}}{26.96 \text{ Btu/lbm}} = 2.06
\]

13-13 The mass fractions of the constituents of a gas mixture are given. The mole fractions of the gas and gas constant are to be determined.

**Properties** The molar masses of CH\(_4\) and CO\(_2\) are 16.0 and 44.0 kg/kmol, respectively (Table A-1)

**Analysis** For convenience, consider 100 kg of the mixture. Then the number of moles of each component and the total number of moles are

\[
m_{\text{CH}_4} = 75 \text{ kg} \quad \rightarrow \quad N_{\text{CH}_4} = \frac{m_{\text{CH}_4}}{M_{\text{CH}_4}} = \frac{75 \text{ kg}}{16 \text{ kg/kmol}} = 4.688 \text{ kmol}
\]

\[
m_{\text{CO}_2} = 25 \text{ kg} \quad \rightarrow \quad N_{\text{CO}_2} = \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}} = \frac{25 \text{ kg}}{44 \text{ kg/kmol}} = 0.568 \text{ kmol}
\]

\[
N_m = N_{\text{CH}_4} + N_{\text{CO}_2} = 4.688 \text{ kmol} + 0.568 \text{ kmol} = 5.256 \text{ kmol}
\]

Then the mole fraction of each component becomes

\[
y_{\text{CH}_4} = \frac{N_{\text{CH}_4}}{N_m} = \frac{4.688 \text{ kmol}}{5.256 \text{ kmol}} = 0.892 \text{ or } 89.2\%
\]

\[
y_{\text{CO}_2} = \frac{N_{\text{CO}_2}}{N_m} = \frac{0.568 \text{ kmol}}{5.256 \text{ kmol}} = 0.108 \text{ or } 10.8\%
\]

The molar mass and the gas constant of the mixture are determined from their definitions,

\[
M_m = \frac{m_m}{N_m} = \frac{100 \text{ kg}}{5.256 \text{ kmol}} = 19.03 \text{ kg/kmol}
\]

and

\[
R_m = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmol} \cdot \text{K}}{19.03 \text{ kg/kmol}} = 0.437 \text{ kJ/kg} \cdot \text{K}
\]

13-33 The masses, temperatures, and pressures of two gases contained in two tanks connected to each other are given. The valve connecting the tanks is opened and the final temperature is measured. The volume of each tank and the final pressure are to be determined.
**Assumptions** Under specified conditions both N\textsubscript{2} and O\textsubscript{2} can be treated as ideal gases, and the mixture as an ideal gas mixture.

**Properties** The molar masses of N\textsubscript{2} and O\textsubscript{2} are 28.0 and 32.0 kg/kmol, respectively. The gas constants of N\textsubscript{2} and O\textsubscript{2} are 0.2968 and 0.2598 kPa·m\textsuperscript{3}/kg·K, respectively (Table A-1).

**Analysis** The volumes of the tanks are

\[ V_{N_2} = \frac{mRT}{P} = \frac{(1 \text{ kg})(0.2968 \text{ kPa·m}^3/\text{kg·K})(298 \text{ K})}{300 \text{ kPa}} = 0.295 \text{ m}^3 \]

\[ V_{O_2} = \frac{mRT}{P} = \frac{(3 \text{ kg})(0.2598 \text{ kPa·m}^3/\text{kg·K})(298 \text{ K})}{500 \text{ kPa}} = 0.465 \text{ m}^3 \]

\[ V_{\text{total}} = V_{N_2} + V_{O_2} = 0.295 \text{ m}^3 + 0.465 \text{ m}^3 = 0.76 \text{ m}^3 \]

Also,

\[ N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{1 \text{ kg}}{28 \text{ kg/kmol}} = 0.03571 \text{ kmol} \]

\[ N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{3 \text{ kg}}{32 \text{ kg/kmol}} = 0.09375 \text{ kmol} \]

\[ N_m = N_{N_2} + N_{O_2} = 0.03571 \text{ kmol} + 0.09375 \text{ kmol} = 0.1295 \text{ kmol} \]

Thus,

\[ P_m = \left( \frac{NR_mT}{V} \right)_m = \frac{(0.1295 \text{ kmol})(8.314 \text{ kPa·m}^3/\text{kmol·K})(298 \text{ K})}{0.76 \text{ m}^3} = 422.2 \text{ kPa} \]
Heat is transferred to a gas mixture contained in a piston cylinder device. The initial state and the final temperature are given. The heat transfer is to be determined for the ideal gas and non-ideal gas cases.

**Properties** The molar masses of $\text{H}_2$ and $\text{N}_2$ are 2.0, and 28.0 kg/kmol. (Table A-1).

**Analysis** From the energy balance relation,

\[ E_{\text{in}} - E_{\text{out}} = \Delta E \]

\[ Q_{\text{in}} - W_{\text{b, out}} = \Delta U \]

\[ Q_{\text{in}} = \Delta H = \Delta H_{\text{H}_2} + \Delta H_{\text{N}_2} = N_{\text{H}_2} \left( \bar{h}_2 - \bar{h}_1 \right)_{\text{H}_2} + N_{\text{N}_2} \left( \bar{h}_2 - \bar{h}_1 \right)_{\text{N}_2} \]

since $W_b$ and $\Delta U$ combine into $\Delta H$ for quasi-equilibrium constant pressure processes

\[ N_{\text{H}_2} = \frac{m_{\text{H}_2}}{M_{\text{H}_2}} = \frac{6 \text{ kg}}{2 \text{ kg/kmol}} = 3 \text{ kmol} \]

\[ N_{\text{N}_2} = \frac{m_{\text{N}_2}}{M_{\text{N}_2}} = \frac{21 \text{ kg}}{28 \text{ kg/kmol}} = 0.75 \text{ kmol} \]

(a) Assuming ideal gas behavior, the inlet and exit enthalpies of $\text{H}_2$ and $\text{N}_2$ are determined from the ideal gas tables to be

\[ \bar{h}_1 = \bar{h}_{\text{H}_2,160 \text{ K}} = 4,535.4 \text{ kJ/kmol}, \quad \bar{h}_2 = \bar{h}_{\text{H}_2,200 \text{ K}} = 5,669.2 \text{ kJ/kmol} \]

\[ \bar{h}_1 = \bar{h}_{\text{N}_2,160 \text{ K}} = 4,648 \text{ kJ/kmol}, \quad \bar{h}_2 = \bar{h}_{\text{N}_2,200 \text{ K}} = 5,810 \text{ kJ/kmol} \]

Thus, \[ Q_{\text{ideal}} = 3 \times (5,669.2 - 4,535.4) + 0.75 \times (5,810 - 4,648) = \textbf{4273 kJ} \]

(b) Using Amagat's law and the generalized enthalpy departure chart, the enthalpy change of each gas is determined to be

\[ T_{R_1,\text{H}_2} = \frac{T_{m,1}}{T_{c,\text{H}_2}} = \frac{160}{33.3} = 4.805 \quad \text{(Fig. A-29)} \]

\[ H_2: \quad P_{R_1,\text{H}_2} = P_{R_2,\text{H}_2} = \frac{P_m}{P_{c,\text{H}_2}} = \frac{5}{1.30} = 3.846 \quad Z_{h_1} \approx 0 \]

\[ T_{R_2,\text{H}_2} = \frac{T_{m,2}}{T_{c,\text{H}_2}} = \frac{200}{33.3} = 6.006 \quad \text{(Fig. A-29)} \]

Thus $\text{H}_2$ can be treated as an ideal gas during this process.

\[ T_{R_1,\text{N}_2} = \frac{T_{m,1}}{T_{c,\text{N}_2}} = \frac{160}{126.2} = 1.27 \quad \text{(Fig. A-29)} \]

\[ N_2: \quad P_{R_1,\text{N}_2} = P_{R_2,\text{N}_2} = \frac{P_m}{P_{c,\text{N}_2}} = \frac{5}{3.39} = 1.47 \quad Z_{h_2} \approx 0 \]

\[ T_{R_2,\text{N}_2} = \frac{T_{m,2}}{T_{c,\text{N}_2}} = \frac{200}{126.2} = 1.58 \]

Therefore,

\[ \left( \bar{h}_2 - \bar{h}_1 \right)_{\text{H}_2} = \left( \bar{h}_2 - \bar{h}_1 \right)_{\text{H}_2,\text{ideal}} = 5,669.2 - 4,535.4 = 1,133.8 \text{ kJ/kmol} \]

\[ \left( \bar{h}_2 - \bar{h}_1 \right)_{\text{N}_2} = R_g T_c \left( Z_{h_1} - Z_{h_2} \right) + \left( \bar{h}_2 - \bar{h}_1 \right)_{\text{ideal}} \]

\[ = (8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(126.2 \text{ K})(1.3 - 0.7) + (5,810 - 4,648) \text{ kJ/kmol} = 1,791.5 \text{ kJ/kmol} \]

Substituting, \[ Q_{\text{in}} = (3 \text{ kmol})(1,133.8 \text{ kJ/kmol}) + (0.75 \text{ kmol})(1,791.5 \text{ kJ/kmol}) = \textbf{4745 kJ} \]
The masses of components of a gas mixture are given. This mixture is expanded in an adiabatic, steady-flow turbine of specified isentropic efficiency. The second law efficiency and the exergy destruction during this expansion process are to be determined.

**Assumptions** All gases will be modeled as ideal gases with constant specific heats.

**Properties** The molar masses of O₂, CO₂, and He are 32.0, 44.0, and 4.0 kg/kmol, respectively (Table A-1). The constant-pressure specific heats of these gases at room temperature are 0.918, 0.846, and 5.1926 kJ/kg·K, respectively (Table A-2a).

**Analysis** The total mass of the mixture is

\[ m_m = m_{O_2} + m_{CO_2} + m_{He} = 1.1 + 1 + 0.5 = 1.6 \text{ kg} \]

The mole numbers of each component are

\[ N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{0.1 \text{ kg}}{32 \text{ kg/kmol}} = 0.003125 \text{ kmol} \]
\[ N_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{1 \text{ kg}}{44 \text{ kg/kmol}} = 0.02273 \text{ kmol} \]
\[ N_{He} = \frac{m_{He}}{M_{He}} = \frac{0.5 \text{ kg}}{4 \text{ kg/kmol}} = 0.125 \text{ kmol} \]

The mole number of the mixture is

\[ N_m = N_{O_2} + N_{CO_2} + N_{He} = 0.003125 + 0.02273 + 0.125 = 0.15086 \text{ kmol} \]

The apparent molecular weight of the mixture is

\[ M_m = \frac{m_m}{N_m} = \frac{1.6 \text{ kg}}{0.15086 \text{ kmol}} = 10.61 \text{ kg/kmol} \]

The apparent gas constant of the mixture is

\[ R = \frac{R_n}{M_m} = \frac{8.314 \text{ kJ/kmol} \cdot \text{K}}{10.61 \text{ kg/kmol}} = 0.7836 \text{ kJ/kg} \cdot \text{K} \]

The mass fractions are

\[ mf_{O_2} = \frac{m_{O_2}}{m_m} = \frac{0.1 \text{ kg}}{1.6 \text{ kg}} = 0.0625 \]
\[ mf_{CO_2} = \frac{m_{CO_2}}{m_m} = \frac{1 \text{ kg}}{1.6 \text{ kg}} = 0.625 \]
\[ mf_{He} = \frac{m_{He}}{m_m} = \frac{0.5 \text{ kg}}{1.6 \text{ kg}} = 0.3125 \]

The constant-pressure specific heat of the mixture is determined from

\[ c_p = mf_{O_2}c_{p,O_2} + mf_{CO_2}c_{p,CO_2} + mf_{He}c_{p,He} \]
\[ = 0.0625 \times 0.918 + 0.625 \times 0.846 + 0.3125 \times 5.1926 \]
\[ = 2.209 \text{ kJ/kg} \cdot \text{K} \]

Then the constant-volume specific heat is

\[ c_v = c_p - R = 2.209 - 0.7836 = 1.425 \text{ kJ/kg} \cdot \text{K} \]

The specific heat ratio is

\[ k = \frac{c_p}{c_v} = \frac{2.209}{1.425} = 1.550 \]

The temperature at the end of the expansion for the isentropic process is
Using the definition of turbine isentropic efficiency, the actual outlet temperature is

\[ T_2 = T_1 - \eta_{turb} (T_1 - T_{2s}) = (600 \text{ K}) - (0.90)(600 - 265) = 299 \text{ K} \]

The entropy change of the gas mixture is

\[ s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = (2.209) \ln \frac{299}{600} - (0.7836) \ln \frac{100}{1000} = 0.2658 \text{ kJ/kg} \cdot \text{K} \]

The actual work produced is

\[ w_{\text{out}} = h_1 - h_2 = c_p (T_1 - T_2) = (2.209 \text{ kJ/kg} \cdot \text{K})(600 - 299) \text{ K} = 665 \text{ kJ/kg} \]

The reversible work output is

\[ w_{\text{rev, out}} = h_1 - h_0 - T_0 (s_1 - s_2) = 665 \text{ kJ/kg} - (298 \text{ K})(-0.2658 \text{ kJ/kg} \cdot \text{K}) = 744 \text{ kJ/kg} \]

The second-law efficiency and the exergy destruction are then

\[ \eta_{II} = \frac{w_{\text{out}}}{w_{\text{rev, out}}} = \frac{665}{744} = 0.894 \]

\[ x_{\text{dest}} = w_{\text{rev, out}} - w_{\text{out}} = 744 - 665 = 79 \text{ kJ/kg} \]